Abstract. This work is dedicated to the specification of correction factors for various methods to determine local energy dissipation rates in the tank. A measuring PIV system to determine velocity field and liquid turbulence field in the tank cross section versus its axis at varying rotation frequencies of the impeller was applied in the investigations. Self-aspirating disk impellers without gas dispersion (the onset of self-aspiration) were used for mixing. The point of reference for the calculations was the mean energy dissipation rate in the tank determined on the basis of mixing power. The calculations revealed that the best results were obtained using the methods based on dimensional equation and direct method with the application of Smagorinsky filtration. It was found, however, that corrections depended additionally on the scale of turbulence and the distance between measured vectors being a result of the employed PIV method.

Keywords: energy dissipation rate, PIV, self-aspirating disk impeller

1. INTRODUCTION

Energy dissipation rate is one of the most important parameters that, among other factors, determine local mixing intensity. It affects transport coefficients as well as gas bubbles dispersed in a mixer. This parameter is related to velocity pulsations and it is often determined by the simple relation [1,2,3]

\[ \varepsilon = C \cdot \frac{\bar{u}^3}{L} \]  

where: \( \bar{u} \) – mean velocity pulsation in reference to RMS [m/s], \( C \) – numerical coefficient, \( L \) – linear dimension [m]. According to the theory, a linear dimension should be the size of the biggest vortices in the tank (integral vortex scale). However, usually it is not known and the impeller diameter is substituted as the linear dimension. This is possible due to the fact that irrespective of the assumed linear dimension the value of coefficient \( C \) should be determined experimentally.

Another method to determine the value of \( \varepsilon \), when the field of instantaneous velocity pulsations \( u \) is known, is to calculate it directly from the equation [4]

\[ \varepsilon = \frac{1}{2} \cdot v \cdot \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 = v \cdot \frac{\partial u_j}{\partial x_i} \cdot \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  

where subscripts \( i \) and \( j \) denote directions in the Cartesian coordinate system \( i = 1,2,3, j = 1,2,3 \). This requires the knowledge of velocity pulsation fields in three directions. However, in the case of isotropic turbulence equation (2) can be transformed [1,5] to form (3)
where \( u_x \) and \( u_y \) are the velocity pulsations in directions \( x \) and \( y \), respectively, perpendicular to each other. Literature data [1,6,7] show that equation (2) gives correct results at spatial resolutions close to the Kolmogorov scale. That is why in other cases it is suggested to introduce correction factors for the value of \( \varepsilon \). The simplest one can be a multiplier whose value depends on mean linear Kolmogorov scale [8]. However, such a simple correction is not always sufficient and corrections based on e.g. turbulence models are introduced. According to the literature [3,9], good results are provided by filtration based on the Smagorinski model

\[
\varepsilon = (C_s \cdot \Delta t)^2 \cdot \left[ \frac{1}{2} \cdot \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \left[ \frac{1}{2} \cdot \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]^{3/2}
\]

(4)

but the obtained values are affected by the values of the so-called Smagorinski constant (usually \( C_s = 0.17 \), but there are also values ranging from 0.11 to 0.21 [3,10]).

The major aim of this study is to determine coefficient \( C \) in equation (1) for self-aspirating disk impeller on the basis of velocity fields obtained by the PIV method. Another target of the study is to investigate the possibility of a simple correction of equation (3) and to apply the correction factor based on the Smagorinski model to calculate energy dissipation rates from equation (3).

2. EXPERIMENTAL

The experiments were carried out in a glass tank with a flat bottom of diameter \( T = 292 \) mm equipped with four standard baffles (Fig. 1). Up to the height \( H = T \) the tank was filled with distilled water to which glass tracer beads of diameter 10 \( \mu \)m were added. The self-aspirating disk impeller of diameter \( D = 125 \) mm was placed at the height \( h_m = 62 \) mm and was operating without gas dispersion at rotation frequencies \( N = 5, 6, 7, 8, 9, 10 \) and \( 11 \) s\(^{-1} \).

Fig. 1. Schematic of a measuring set-up and control volume

A camera of the PIV measuring system with the resolution of 2048×2048 pixels equipped with a Zeiss Jena Vario-Pancolar 35-70 mm f/2.4-f/2.7 lens was mounted below the tank bottom. By moving the camera and changing lens focal length the imaging field was set so as to cover over \( 1/4 \) of the tank cross section with two subsequent baffles visible. A laser light knife about 0.5 mm thick lit the tank in the plane parallel to the tank bottom. 100 pairs of images were recorded at the heights \( h_L = 18, 35, 45, 52, 62, 71, 80, 95, 110, 125, 145, 165, \)
185, 205, 225, 245 and 265 mm. The images were processed using the DaVis 7.2 program. A two-pass data processing was applied. In the first pass dimensions of the interrogation area were 64 px×64 px, and in the second one 32 px×32 px. The final value of the interrogation area was specified on the basis of literature data [11] and our own experiments [12]. Spatial resolution was around 0.083 mm/px. To determine velocity fields an overlay of measuring fields of the values 0%, 25%, 50% and 75% was used. This gives distances between velocity vectors \( \Delta l = 2.656, 1.982, 1.324 \) and 0.665 mm.

3. RESULTS AND DISCUSSION

For each measuring series mean local velocity pulsations for each control volume were calculated from the vector length under the assumption of isotropic turbulence. Next, weighted average \( \frac{u^3}{D} \) was calculated with the values of weights equal to the share of control volumes and the values of coefficient \( C \) were calculated (Fig. 2a).

Based on the derivatives of instantaneous velocity pulsations, energy dissipation rates in control volumes were calculated from equations (3) and (4). On this basis weighted averages of energy dissipation rates for the whole tank were determined taking as a weight the ratio of control volume to liquid volume in the tank. Results are shown in Figure 2b.

![Fig. 2. Comparison of mean values \( \varepsilon \) before (a) and after correction (b)](image)

As follows from the analysis of Figure 2, there is a strong dependence of the results on the distance between velocity vectors. The value of coefficient \( C \) in equation (1) for small energy dissipation is close to 1. For higher rotation frequencies the deviation from this value grows. An increase of coefficient \( C \) is most probably a result of assuming in equation (1) the impeller diameter as a linear dimension and not the total scale of vortices whose value may change depending on rotation frequency of the impeller. Equation (3) gives the smallest values of \( \varepsilon \), while values obtained from equation (4) are the biggest. On the basis of data analysis (Fig. 3a and 3b) the dependence of coefficient \( C \) and coefficients \( C_1 \) and \( C_2 \) correcting equations (3) and (4) on the dimension of measuring field \( \Delta l \) (i.e. the distance between velocity vectors) and average for the whole tank in the Kolmogorov scale \( \langle l_k \rangle \) was determined. The values of \( \langle l_k \rangle \) were specified based on mixing power for the whole tank; they were equal to 0.050, 0.044, 0.039, 0.032, 0.030 and 0.028 mm for growing rotation frequencies of the impeller.

\[
C = 0.95 + 1.386 \cdot 10^{-12} \cdot \left( \frac{D}{\Delta l} \right)^{3.25} \cdot \left( \frac{\Delta l}{\langle l_k \rangle} \right)^3
\]  
\[
C_1 = 0.01045 \cdot \left( \frac{\Delta l}{\langle l_k \rangle} \right)^{1.852}
\]  
\[
C_2 = 0.00321 \cdot \left( \frac{D}{\Delta l} \right)^{0.432} \cdot \left( \frac{\Delta l}{\langle l_k \rangle} \right) - 0.326
\]
After considering corrections calculated from equations (6), (7) and (8) only in some cases the weighted average for $\varepsilon$ differs from the value calculated from mixing power. Since this compatibility can be achieved at different distributions of local values, further on an analysis of $\varepsilon$ distributions on several levels will be made. Because most energy is dispersed on the impeller suspension height, it is most important to compare the maps of $\varepsilon$ distributions at height $h_1 = 62$ mm. The contour plots for three rotation frequencies at 25% overlay are shown in Figures 3 to 5.

![Fig. 3. Distributions of $\varepsilon$ on the height of impeller suspension calculated from eq. (1) after correction](image)

![Fig. 4. Distributions of $\varepsilon$ on the height of impeller suspension calculated from eq. (3) after correction](image)

![Fig. 5. Distributions of $\varepsilon$ on the height of impeller suspension calculated from eq. (4) after correction](image)

From the comparison of maps for the same rotation frequencies it follows that the values of $\varepsilon$ obtained from equation (3) and corrected by coefficient $C_1$ are about three times smaller near the impeller than those calculated from equation (1). Near the wall differences in the value of $\varepsilon$ calculated by means of the two compared methods are insignificant. The $\varepsilon$ values calculated from equation (4) and corrected by multiplier $C_2$ to achieve compatibility with energy dissipation rate determined on the basis of mixing power reveal intermediate values.

Different relations are observed for the region below the height of impeller suspension (Figures 6, 7 and 8). 17 mm below the impeller level the maximum energy dissipation rates calculated from corrected relations (1) and (3) become equal, while relation (4) for the radius equal to the impeller radius reveals the highest values of $\varepsilon$. Outside this region, relation (3)
gives the highest values of $\varepsilon$, while the other two relations provide similar energy dissipation rates. A general conclusion can be drawn from these results that in places where local values of the Kolmogorov scale are smaller than the mean value of the whole tank, the correction of values calculated from relation (3) should be stronger (higher value of the multiplier than it follows from equation (7)), while when the value of Kolmogorov scale is smaller than the mean value, the correction should be weaker. Hence, in equation (7) there should be a local value of the Kolmogorov scale which is not always known. Therefore, this method of correction allows us to calculate correctly only the mean value of $\varepsilon$ for the whole tank, while distributions of local values, which are significant for the analysis of the way in which gas is dispersed by the tested impeller, will be burdened with significant error.

4. CONCLUSIONS

The calculations show that in a hypothetical cylinder of diameter 200 mm and height 30 mm, which is ca. 4.8% of the liquid volume, about 2/3 of energy supplied to the tank is dissipated. Below and above the impeller, the area of the highest energy dissipation rate is behind the baffle; between the baffles energy dissipation rates attain the smallest value.
Dimensional equation (1) enables quick determination of $\varepsilon$ value in the PIV measurements. However, when mean impeller diameter is assumed to be a linear dimension, the value of coefficient $C$ depends on rotation frequency of the impeller (indirectly, on linear turbulence scales).

A simple correction of equation (3) based on mean linear Kolmogorov scale does not bring about correct values of local energy dissipation rates. Filtration based on the Smagorinski model requires additional corrections to achieve compatibility with mean values obtained on the basis of mixing power.

The biggest differences in the $\varepsilon$ values calculated from corrected relations (1) and (4) occur near the tips of impeller blades where the assumption of isotropic turbulence is not satisfied.

Most energy is dissipated near the impeller (about 2/3 in the cylinder of diameter 200 mm and height 30 mm). Beside this region, the value of $\varepsilon$ changes quickly with the distance from the rotating impeller blades.

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4. REFERENCES